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THE CONDITION FOR ASSESSING THE ADEQUACY OF MODELS FOR PROTECTING STATE INFRASTRUCTURE OBJECTS FROM AIR BLOWS DURING HOSTILITIES

To control the adequacy of the mathematical models for planning and managing the defense against air blows on the covered state infrastructure objects by the most effective means – groupings of surface-to-air missile systems during military operations, an analytical description of the internal law of air defence battles was found. The presence of the noted law in the developed mathematical models can be a criterion for their adequacy to real processes of combat operations and a necessary condition for the models applicability in practice.

Keywords: *mathematical models, air defence battle, covered objects, adequacy.*

Formulation of the problem. One of the public administration tasks during the period of hostilities is to create conditions for protecting infrastructure objects from being hit by enemy aircraft and other means of air attack throughout the state.

At the same time, surface-to-air missile (SAM) troops have the greatest efficiency in solving the problems of covering state objects from air blows.

The armament of each subunit includes a surface-to-air missile system of a specific type with a stock of anti-aircraft guided missiles, means of radar reconnaissance and identification of aircraft nationality, means of receiving target designation and communication means.

Infrastructure objects of the state can be located at a considerable distance from each other, which, in the conditions of a limited number of SAM systems, does not allow the creation of SAM systems zonal groupings and leads to the need to form object SAM systems groupings to protect groups and individual objects.

The composition of each SAM grouping is often determined by the need to create a continuous zone of fire on the approaches to covering objects up to the line of tasks performance by an air enemy.

In turn, the air adversary, as a rule, has some information about the combat formation of the SAM grouping, and plans the parameters of the air blow – the composition, combat formation of means of air attack (MAA) and the procedure for overcoming the defended airspace, using radar counteraction and SAM systems fire damage with the entry into the SAM subunits zones of fire and without it.

However, in the course of each parties tasks carrying out, their plan of action is transformed into a sequence of random fire contacts, unpredictable in terms of time and results.

As a result, the planned nature of the infrastructure objects protection systems preparation, which includes determining the types, composition of SAM groups, their deployment on the ground and all types of their support, requires taking into account the characteristics of random development and random results of fire contacts.

In many cases, such features include the actions of enemy means of air attack (MAA) in groups with a previously unknown composition of groups, the unpredictability (randomness) of the start and end times of each fire contact of the sides' weapons, the possibility of hitting the weapons of each side during fire contacts, the effect is not full availability of the enemy MAA to individual SAM groups due to the limited size of the fire zones and the placement of SAM units on the ground, as well as other features.

As a result, the implementation of the noted features becomes the reason for the lack of guarantees in solving the problems of protecting state infrastructure objects, and there is a general problem of planning and managing the protection of groups and individual infrastructure objects, taking into account the noted features of its implementation.

One of the directions for solving the problem can be the development of the models set for the process of performing tasks to protect infrastructure objects that allow taking into account the noted and other features.The development of such a models' complex is beyond the scope of this consideration.

However, every time the appearance of the corresponding models is faced with an independent problem of assessing their adequacy to the real process and their applicability in practice.

The solution of such a key problem is necessary condition for success in solving the noted general problem of taking into account the peculiarities of planning and managing the protection of state infrastructure objects during hostilities, which determines the relevance of the research topic.

The recent research and publications analysis. In well-known publications on the development of infrastructure objects covering from air strikes process models, the issue of assessing models' adequacy was not considered. At the same time, according to the composition of the mathematical apparatus used, the known works can be divided into several categories.

Thus, when constructing models for assessing the effectiveness of air defense and missile defense of ground objects [1–4] and a grouping of surface ships [5], the mathematical apparatus of queuing theory [1, 5], game theory [2], Petri nets [4], as well as the idea of heterogeneous networks [3] were used.

At the same time, the authors intuitively believe that the chosen mathematical apparatus corresponds to the processes of air defense battle of AAM grouping. However, the listed mathematical methods do not take into account the main factor of air defense battle – the possibility of SAM systems damage, which makes the noted models devoid of adequacy to real combat processes.

The purpose of the article is to search for and analytically describe such an internal property of covering processes from air blows the state infrastructure objects by the most effective means – SAM groupings, which can be used to control models adequacy of covered object protection planning and management.

The main material's presentation. Surface-to-air missile troops have the highest effectiveness in solving the tasks of state facilities air defence from air blows. SAM subunits may be armed with SAM systems of different ranges, radar reconnaissance and aircraft identification, targeting and communications equipment.

An air enemy, as a rule, plans his actions using known means of radar and fire countermeasures and methods of protection against SAM fire, seeks to carry out his tasks either without entering the SAM grouping's fire zone, or with a minimum stay in such a zone.

For each SAM subunit, the processes of air defence battle include the moments of means of air attack detection, the moments of the beginning and end of the firing cycle, which are not known in advance and have the properties of continuous random variables. The result of each fire contact of SAM system with an air target is also not known in advance (is random) and may include the following events: an enemy aircraft downed (with a probability of $P_{d.MAA}$), an enemy aircraft not downed (with a probability of $1 - P_{d.MAA}$), a SAM system destroyed (with probability of $P_{d.SAM}$), a SAM system not destroyed (with probability of $1 - P_{d.SAM}$), combinations of all marked options.

As a result, the processes of fire contacts of each subunit and the entire group of SAM systems turn out to be random in time and have random results, which makes it difficult to create models that are adequate enough for a real battle and suitable for making informed decisions on the organization of air defense.

One of the peculiarities of the models development for planning and organizing air defense is the need for the mathematical apparatus correct choice that is adequate to the laws of random variables distribution, as well as the creation of conditions for the possibility of controlling the adequacy of such models for real air defence battle's processes.

Unfortunately, when developing the well-known [1, 2] and other models, the noted peculiarities and the processes of assessing the adequacy of models for real battle were not considered.

At the same time, the noted peculiarities of each SAM system's fire contact with means of air attack make it possible to establish some non-obvious properties of the SAM system grouping's air defence battle.

Let's denote the initial number of SAM systems in the grouping by the symbol n_0, the mathematical expectations of fire contacts number made during the air defence battle of the SAM system grouping, the number of destroyed SAM systems and the number of downed MAA by the symbols n_{fc}, $n_{d.SAM}$ and $n_{d.MAA}$, respectively. In this case, at any current time t of the battle, are true the relations:

$$\left.\begin{aligned} n_{d.SAM}(t) &= n_{fc}(t)\cdot P_{d.SAM}, \\ n_{d.MAA}(t) &= n_{fc}(t)\cdot P_{d.MAA}. \end{aligned}\right\} \qquad (1)$$

Let's also assume a situation in which there is the possibility of an unlimited number of MAA in a blow and an unlimited ammunition load of anti-aircraft guided missiles in each SAM subunit. In this case, over some time, all n_0 subunits of the

SAM group will be hit, the battle will stop, the mathematical expectations of the battles number and the number of downed MAA will reach their maximum values $n_{fc\infty}$ and $n_{d.MAA\infty}$:

$$\left.\begin{aligned} \lim_{t\to\infty} n_{d.SAM}(t) = n_0,\ \lim_{t\to\infty} n_{fc}(t) = n_{fc\infty},\\ \lim_{t\to\infty} n_{d.MAA}(t) = n_{d.MAA\infty} \end{aligned}\right\} \quad (2)$$

which can be found from (1):

$$\left.\begin{aligned} n_0 = n_{fc\infty}\cdot P_{d.SAM},\quad n_{fc\infty} = \frac{n_0}{P_{d.SAM}},\\ n_{d.MAA\infty} = n_{fc\infty}\cdot P_{d.MAA} = n_0\cdot\frac{P_{d.MAA}}{P_{d.SAM}}. \end{aligned}\right\} \quad (3)$$

In this case, we can proceed to the relative values of the mathematical expectations of fire contacts number N_{fc} carried out during the air defence battle of the SAM grouping, the number of downed MAA $N_{d.MAA}$ and the number $N_{d.SAM}$ of the destroyed SAM systems:

$$\left.\begin{aligned} N_{fc}(t) = \frac{n_{fc}(t)}{n_{fc\infty}},\quad N_{d.SAM}(t) = \frac{n_{d.SAM}(t)}{n_0},\\ N_{d.MAA}(t) = \frac{n_{d.MAA}(t)}{n_{d.MAA\infty}}. \end{aligned}\right\} \quad (4)$$

To establish the relationship between the values in (4), we divide the left and right parts of the first formula in (1) by the initial number of SAM systems n_0 in the SAM group, and take into account equalities (3) and (4), we get:

$$\left.\begin{aligned} \frac{n_{d.SAM}(t)}{n_0} = \frac{n_{fc}(t)\cdot P_{d.SAM}}{n_0} = \frac{n_{fc}(t)}{n_0/P_{d.SAM}} =\\ = \frac{n_{fc}(t)}{n_{fc\infty}},\quad N_{d.SAM}(t) = N_{fc}(t). \end{aligned}\right\} \quad (5)$$

Thus, at any moment of the battle, the relative values of the mathematical expectations of destroyed SAM systems number in the grouping and fire contacts number turn out to be equal to each other.

Further, on the right side of the first equality in (5), we multiply the numerator and denominator of the fraction by the probability $P_{d.MAA}$ of the MAA destruction during one fire contact, we get:

$$\left.\begin{aligned} \frac{n_{d.SAM}(t)}{n_0} = \frac{n_{fc}(t)\cdot P_{d.MAA}}{n_{fc\infty}\cdot P_{d.MAA}} = \frac{N_{d.MAA}}{N_{d.MAA\infty}},\\ N_{d.SAM}(t) = N_{d.MAA}(t). \end{aligned}\right\} \quad (6)$$

Equations (5) and (6) imply the equality:

$$N_{d.SAM}(t) = N_{d.MAA}(t) = N_{fc}(t). \quad (7)$$

Thus, the key peculiarity of the models development for planning and organizing air defense is the need to ensure in the model equality (7) of the relative values of the mathematical expectations for the number of destroyed SAM systems in the SAM grouping, the number of downed MAA and the number of fire contacts at any time of the battle.

The presence of regularity (7) in the developed models of air defence battle may testify in favor of their adequacy to the described battle processes. On the other hand, the absence of regularity (7) in the battle model calls into question the adequacy of the model.

Conclusions

In the article was found an analytical description of the internal key property of performing tasks processes by the most effective means – groupings of SAM systems for covering state infrastructure objects from air blows.

The found property can be used to control the adequacy of the mathematical models complex for planning and management the protection of state infrastructure objects from air blows.

The materials of the article can be used in the practical activities of the mathematical models and their complexes developers, designed to plan and manage the protection of the covered state objects from air strikes by the most effective means – SAM groupings during military operations.

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УМОВА ОЦІНКИ АДЕКВАТНОСТІ МОДЕЛЕЙ ЗАХИСТУ ОБ'ЄКТІВ ІНФРАСТРУКТУРИ ДЕРЖАВИ ВІД УДАРІВ З ПОВІТРЯ У ХОДІ ВІЙСЬКОВИХ ДІЙ

Одним із завдань державного управління у період воєнних дій є захист об'єктів інфраструктури від ураження авіаційними та іншими засобами повітряного нападу супротивника на всій території держави. Водночас найбільшою ефективністю вирішення завдань прикриття об'єктів держави від ударів з повітря мають угруповання зенітних ракетних військ. Об'єкти інфраструктури держави можуть розміщуватися на значній відстані один від одного, що в умовах обмеженої кількості зенітних ракетних комплексів веде до необхідності формування об'єктових угруповань для захисту груп та окремих об'єктів інфраструктури держави. Зі свого боку повітряний противник, як правило, планує свої дії із застосуванням відомих засобів радіолокаційної і вогневої протидії та методів захисту від ураження вогнем зенітних ракетних комплексів, прагне виконувати свої завдання або без входу в зону вогню угруповання зенітних ракетних комплексів, або з мінімальним часом перебування у такій зоні.

Однак плановий характер підготовки до дій сил і засобів захисту та сил і засобів повітряного нападу в ході виконання завдань кожної зі сторін перетворюється на послідовність випадкових вогневих контактів, що не передбачаються за часом і за результатами.

У результаті плановий характер підготовки захисту об'єктів інфраструктури перетворюється на випадковий процес послідовності вогневих контактів засобів сторін з важко прогнозованими результатами.

За таких умов підготовка та управління захистом об'єктів інфраструктури може бути забезпечена внаслідок розроблення відповідного комплексу математичних моделей. Також розроблення моделей стикається із самостійною проблемою оцінки їхньої адекватності реальному процесу як ключовій умові їх застосування на практиці.

Для контролю адекватності таких моделей у статті знайдено аналітичний опис внутрішнього закону протиповітряних боїв, наявність якого в математичних моделях, що розробляються, може бути критерієм їхньої адекватності реальним процесам бойових дій.

Ключові слова: *математичні моделі, протиповітряний бій, об'єкти, що прикриваються, адекватність.*

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