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MATHEMATICAL MODELING AND ANALYSIS OF THE TECHNOLOGICAL ERROR OF THE MEANS OF MEASURING THE GEOMETRIC CHARACTERISTICS OF THE FIREARMS BORES

The parameters of the optical scheme of a laser triangulation sensor, whose deviation from their nominal (calculated) values will affect the transformation function, are determined. It is shown that the presence of a technological error during the processing of measurement information will be perceived as an apparent change in the measured value with a constant input signal.

Mathematical models of the components of technological error are obtained, which are presented in the form of error limits at the input of the measuring instrument. The expressions for determining the confidence limits of the total technological error, taking into account the correlation between its components, are substantiated.

A quantitative assessment of the technological error for characteristic combinations of weapon parameters is carried out. The results of the quantitative evaluation are presented in the form of graphs of the dependence of the limits of relative and absolute technological error on the increase in the radius of the bore in the measurement range.

Keywords: *mathematical model, bore, technical condition, diagnostic method, laser triangulation sensor, geometric characteristic, technological error, measuring instrument, technical diagnosis, error component.*

Statement of the problem. According to the experience of using firearms, the technical condition of the bore is a significant factor that affects the effectiveness of the fire mission. The technical condition of the bore, as well as any diagnostic object, is determined based on the analysis of the results of measuring its characteristics. Acceptable reliability of conclusions about the technical condition of the bore can be achieved if the measurement error of its geometric characteristics is ensured at a level close to that which corresponds to the production capabilities of firearms manufacturing enterprises.

Traditional means of technical diagnostics of the bore are characterized by insufficient information and efficiency, and sometimes even accuracy, which limits the possibility of implementing high-quality control of the technical condition of firearms in the field [1, 2, 3].

Given the above limitations in the use of known means of technical diagnostics of the bore, the creation of a measuring instrument based on a laser triangulation sensor opens up certain prospects. The principle of measurement is to direct the probing beam to the surface under investigation and receive the reflected radiation, the parameters of which carry information about the deviation of the bore surface point from its nominal position.

However, despite the sufficient degree of study of the triangulation method of determining distances, a set of tasks aimed at studying the potentially achievable accuracy of measuring the geometric characteristics of the bore remains relevant. This will make it possible to determine the possibilities or limits of using a measuring device based on a laser triangulation sensor to obtain diagnostic information.

In general, the total error of a measuring instrument based on a laser triangulation sensor is created by a number of components, the estimation of each of which can be found by mathematical modeling with subsequent quantification for characteristic combinations of device parameters. The method of combining such error components will be determined by the nature and relationship of their sources.

One of the main factors that leads to the appearance of the corresponding error component is the technological factor, namely, the deviation of the parameters of the optical circuit of the laser triangulation sensor from their nominal values due to the imperfection of the technology of manufacturing, assembly, and adjustment of the measuring instrument.

Therefore, when studying the limits of applicability of a laser triangulation tool for measuring the geometric characteristics of the bore, the problem of quantifying the technological component of the error arises. This problem can be solved on the basis of the construction and further analysis of the mathematical model of the technological error of the laser triangulation sensor.

Analysis of recent research and publications. Articles [4, 5] describe in detail the method of measuring the geometric characteristics of the bore using a laser triangulation sensor and propose the design of a measuring instrument.

Approaches to modeling the error components of the sensitive elements of precision measuring instruments based on laser sensors are considered in [6–10].

Sources [11–14] describe an approach to mathematical modeling of the error component of a measuring instrument based on establishing a relationship between the apparent change in the output value from a change in a parameter included in the transformation function due to a certain influencing value.

However, as the results of the analysis of the content of the reviewed sources show, they did not formulate or solve the problem of mathematical modeling of the error of a laser triangulation sensor caused by imperfect manufacturing, assembly, and adjustment of the measuring instrument.

The purpose of the article is to develop a mathematical model and quantify the technological error of a laser triangulation sensor used to measure the geometric characteristics of the firearms bore.

Summary of the main material. At the first stage of the research, we determine the parameters of the optical circuit of the laser triangulation sensor, the deviation of which from their nominal (calculated) values will affect the transformation function. At the next stage, we will obtain mathematical models of the components of the technological error in the form of dependencies of the change in the output value on the influencing value, which will be perceived by the measuring channel of the measuring instrument as the appearance of apparent increases in the measured value. At the final stage, it is advisable to quantify the ranges of variation of the limits of technological error of a laser triangulation sensor for characteristic combinations of measurement conditions.

We analyze the variation of the main parameters of the optical scheme of the laser triangulation sensor (Figure 1), namely:

a – distance from the intersection of the main optical axes of the focusing 1 and receiving 2 lenses to the optical center of the receiving lens;

b – distance from the optical center of the receiving lens to the surface of the photosensitive detector 3;

α – angle between the main optical axes of the focusing and receiving lenses;

β – angle between the main optical axis of the receiving lens and the surface of the photosensitive detector.

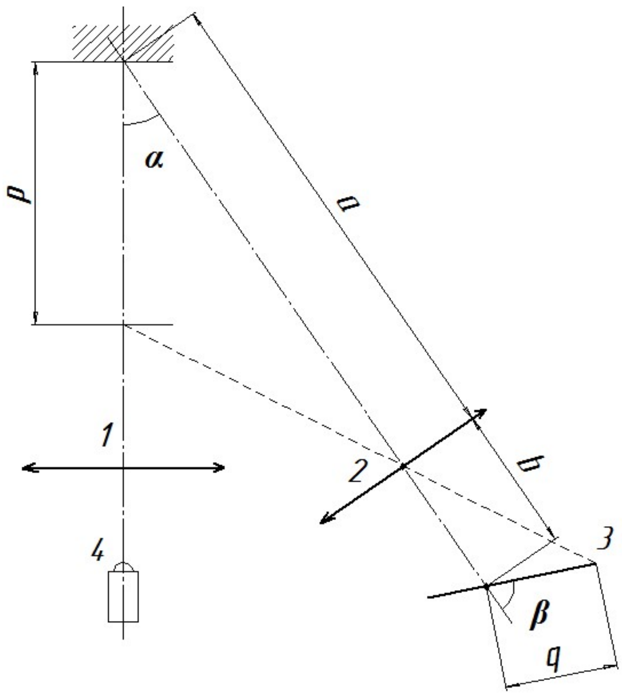

Figure 1 – Optical scheme of the laser triangulation sensor

Also in Figure 1 other symbols are used to indicate:

p – input (measured) value of the laser triangulation sensor (increase in the radius of the bore);

q – output value of the laser triangulation sensor (increase in the coordinate of the light spot on the surface of the photosensitive detector);

4 – laser radiation source.

In the case of deviations of the above parameters of the optical circuit from their nominal values, the latter acquire the corresponding increments, and the following values:

$$\tilde{a}_i = a + \Delta a_i,\ \tilde{b}_i = b + \Delta b_i,\ \tilde{\alpha}_i = \alpha + \Delta \alpha_i,\ \tilde{\beta}_i = \beta + \Delta \beta_i. \quad (1)$$

The manifestations of technological deviations of each of the parameters of the optical scheme will be considered through their influence on the conversion function of the laser triangulation sensor [4]:

$$p = \frac{q \cdot a \cdot \sin \beta}{b \cdot \sin \alpha - q \cdot \sin(\alpha + \beta)}. \quad (2)$$

Thus, if the parameter a is changed according to expression (1), the changed value will be included in the transformation function of the laser triangulation sensor $\tilde{a}_i$. During the processing of measurement information, this will be perceived as a corresponding apparent change in the measured value with an unchanged input signal:

$$p_{a_i} = \frac{q \cdot \tilde{a}_i \cdot \sin\beta}{b \cdot \sin\alpha - q \cdot \sin(\alpha + \beta)}, \quad (3)$$

where p_{a_i} is the apparent value of the measured value due to a change in the parameter *a*.

Similarly to this approach, taking into account expression (1), we can write:

$$p_{b_i} = \frac{q \cdot a \cdot \sin \beta}{\tilde{b}_i \cdot \sin \alpha - q \cdot \sin(\alpha + \beta)}, \quad (4)$$

$$p_{\alpha_i} = \frac{q \cdot a \cdot \sin \beta}{b \cdot \sin \tilde{\alpha}_i - q \cdot \sin(\tilde{\alpha}_i + \beta)}, \quad (5)$$

$$p_{\beta_i} = \frac{q \cdot a \cdot \sin\tilde{\beta}_i}{b \cdot \sin\alpha - q \cdot \sin(\alpha + \tilde{\beta}_i)}. \quad (6)$$

In addition to the above factors, additional factors include the following:

– deviation ΔO of the optical center of the focusing lens from the calculated position;

– deviation ΔA of the position of the base face of the photosensitive detector in the plane of its location.

For the mathematical modeling of these factors, we will also assume that the presence of deviations ΔO, ΔA will be perceived by the measuring channel of the measuring instrument as the appearance of additional apparent increases $\Delta \alpha_i'$, $\Delta \alpha_i''$ of the angle between the main optical axes of the focusing and receiving lenses with the input signal remaining unchanged. Based on this approach, it can be assumed that each of the two factors under consideration will lead to an apparent change in the parameter α of the optical scheme of the laser triangulation sensor:

$$\tilde{\alpha}_i' = \alpha + \Delta \alpha_i',\ \tilde{\alpha}_i'' = \alpha + \Delta \alpha_i''. \quad (7)$$

The apparent values of the measured quantities $p_{\alpha_i'}$ and $p_{\alpha_i''}$, caused by the increments $\Delta \alpha_i'$ and $\Delta \alpha_i''$, can be written in the following form by analogy with expression (3):

$$p_{\alpha_i'} = \frac{q \cdot a \cdot \sin \beta}{b \cdot \sin \tilde{\alpha}_i' - q \cdot \sin(\tilde{\alpha}_i' + \beta)}, \quad (8)$$

$$p_{\alpha_i''} = \frac{q \cdot a \cdot \sin \beta}{b \cdot \sin \tilde{\alpha}_i'' - q \cdot \sin(\tilde{\alpha}_i'' + \beta)}. \quad (9)$$

We will study the deviation ΔO using the optical scheme of the laser triangulation sensor with additional constructions (Figure 2).

Figure 2 shows the following notations:

O – optical centre of the receiving lens;

O' – position of the optical centre of the focusing lens in case of its deviation;

$\Delta \alpha_i'$ – angle at the vertex *A* of the triangle *OAO'*, which is the increment of the angle α under the influence of the factor under consideration;

$\tilde{\alpha}_i'$ – angle between the main optical axes of the focusing and receiving lenses, taking into account the deviation of the optical centre of the focusing lens from the calculated position.

The length of the segment $OO' = \Delta O$ can be expressed as the tangent of the angle $\Delta \alpha_i'$:

$$tg \Delta \alpha_i' = \frac{\Delta O}{a}. \quad (10)$$

We perform trigonometric transformations and get the expression for determining the angle $\Delta \alpha_i'$:

$$\Delta \alpha_i' = arctg\left(\frac{\Delta O}{a}\right). \quad (11)$$

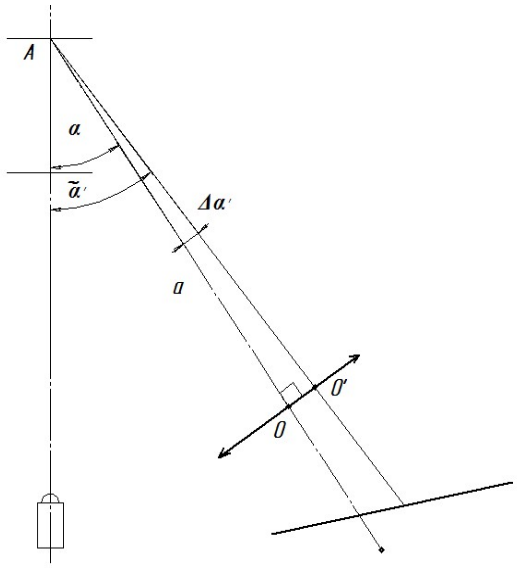

Figure 2 – Optical scheme of the laser triangulation sensor in the presence of deviation of the optical center of the focusing lens

Let us study the deviation ΔA, for which we consider the optical scheme of the laser triangulation sensor with additional constructions (Figure 3).

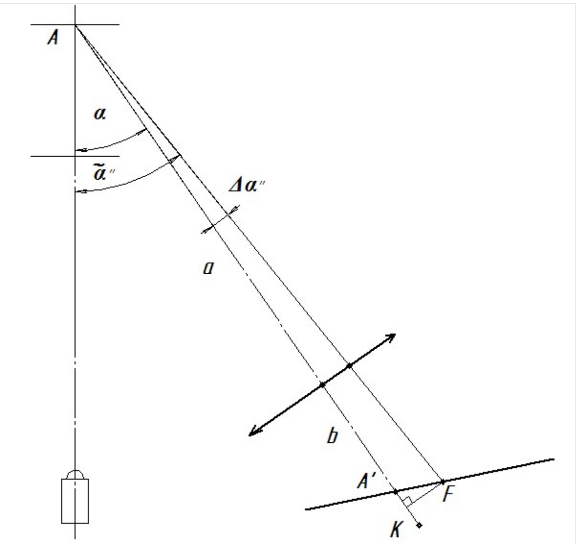

Figure 3 – Optical scheme of the laser triangulation sensor in the presence of a deviation of the base face of the photosensitive detector

Figure 3 shows the following notations:

A' – point of intersection of the main optical axis of the receiving lens with the surface of the photosensitive detector;

$A'F$ – segment whose length is equal to the deviation of the position of the photosensitive detector ΔA;

$\Delta\alpha''_i$ – angle at vertex A of triangle $A'AF'$, which is the increment of angle α due to the deviation of the position of the photosensitive detector;

$\tilde{\alpha}''_i$ – angle between the main optical axes of the focusing and receiving lenses, taking into account the deviation of the position of the photosensitive detector.

Let's lower a perpendicular from point F to the main optical axis of the receiving lens, which will cross it at point K, and consider triangles $A'FK$ and AFK. From the triangle AFK we obtain the expression to determine $\Delta\alpha''_i$:

$$tg\Delta\alpha''_i = FK \,/\, AK\,, \quad (12)$$

where

$$AK = a + b + A'K. \quad (13)$$

From the triangle $A'FK$ we obtain the expression to determine FK and $A'K$:

$$FK = A'F \cdot \sin\beta\,. \quad (14)$$

$$A'K = A'F \cdot \cos\beta\,. \quad (15)$$

Let us substitute expressions (13), (14), (15) into (12) and, given that $A'F = \Delta A$ write:

$$tg\Delta\alpha''_i = \frac{\Delta A \cdot \sin\beta}{a + b + \Delta A \cdot \cos\beta}\,. \quad (16)$$

We perform the proper trigonometric transformations and obtain the expression for determining $\Delta\alpha''$:

$$\Delta\alpha''_i = arctg\left(\frac{\Delta A \cdot \sin\beta}{a + b + \Delta A \cdot \cos\beta}\right). \quad (17)$$

The apparent change in the measured value due to technological deviations will be considered as the appearance of the corresponding components of the instrumental error of the laser triangulation sensor at its input, which are systematic for one particular sample of the measuring instrument, but vary within certain limits for a set of different samples of the measuring instrument. In this case, according to [15], such systematic errors are considered to change irregularly, remaining within the limits $\Delta p_{a_i}, \Delta p_{b_i}$, $\Delta p_{\alpha_i}, \Delta p_{\beta_i}$, $\Delta p_{\alpha'_i}, \Delta p_{\alpha''_i}$. In the future, when combined, these error components are considered as random variables and are assumed to be uniformly distributed within certain limits.

Let us find the error limits $\Delta p_{a_i}, \Delta p_{b_i}$, $\Delta p_{\alpha_i}, \Delta p_{\beta_i}$, $\Delta p_{\alpha'_i}, \Delta p_{\alpha''_i}$ at the input of the laser triangulation sensor. Taking into account expressions (3)–(6) and (7), they can be represented as follows:

$$\Delta p_{a_i} = p_{a_i} - p, \quad \Delta p_{b_i} = p_{b_i} - p, \quad \Delta p_{\alpha'_i} = p_{\alpha'_i} - p, \quad \Delta p_{\alpha''_i} = p_{\alpha''_i} - p, \tag{18}$$

$$\Delta p_{\alpha_i} = p_{\alpha_i} - p, \qquad \Delta p_{\beta_i} = p_{\beta_i} - p. \tag{19}$$

Using expressions (18) and their corresponding (3), (4), and (1), we obtain expressions for determining the error limits of $\Delta p_{a_i}, \Delta p_{b_i}$:

$$\Delta p_{a_i} = \frac{\Delta a_i \cdot q \cdot \sin \beta}{b \cdot \sin \alpha - q \cdot \sin(\alpha + \beta)}, \tag{20}$$

$$\Delta p_{b_i} = \frac{-\Delta b_i \cdot a \cdot q \cdot \sin\alpha \cdot \sin\beta}{(b \cdot \sin\alpha - q \cdot \sin(\alpha+\beta))^2 + \Delta b_i \cdot \sin\alpha \cdot (b \cdot \sin\alpha - q \cdot \sin(\alpha+\beta))}. \tag{21}$$

Using expressions (19) and their corresponding (18) and (1), we obtain expressions for determining the error limits of $\Delta p_{\alpha_i}, \Delta p_{\beta_i}$:

$$\Delta p_{\alpha_i} = a \cdot q \cdot \sin\beta \cdot \frac{b \cdot (\sin\alpha - \sin(\alpha + \Delta\alpha_i)) + q \cdot (\sin(\alpha + \Delta\alpha_i + \beta) - \sin(\alpha+\beta))}{(b \cdot \sin(\alpha + \Delta\alpha_i) - q \cdot \sin(\alpha + \Delta\alpha_i + \beta)) \cdot (b \cdot \sin\alpha - q \cdot \sin(\alpha+\beta))}. \tag{22}$$

Apply the formula for the sine of the sum of angles:

$$\sin(\alpha + \Delta\alpha_i) = \sin\alpha \cdot \cos\Delta\alpha_i + \cos\alpha \cdot \sin\Delta\alpha_i, \tag{23}$$

$$\sin(\alpha + \beta + \Delta\alpha_i) = \sin(\alpha+\beta) \cdot \cos\Delta\alpha_i + \cos(\alpha+\beta) \cdot \sin\Delta\alpha_i. \tag{24}$$

According to the first important limit, expressions (23) and (24) can be represented as

$$\sin(\alpha + \Delta\alpha_i) = \sin\alpha + \cos\alpha \cdot \Delta\alpha_i, \tag{25}$$

$$\sin(\alpha + \beta + \Delta\alpha_i) = \sin(\alpha+\beta) + \cos(\alpha+\beta) \cdot \Delta\alpha_i. \tag{26}$$

The final expression for Δp_{α_i} is obtained by substituting expression (25) into (22):

$$\Delta p_{\alpha_i} = \frac{a \cdot q \cdot \sin\beta \cdot \Delta\alpha_i \cdot (q \cdot \cos(\alpha+\beta) - b \cdot \cos\alpha)}{(b \cdot (\sin\alpha + \cos\alpha \cdot \Delta\alpha_i) - q \cdot (\sin(\alpha+\beta) + \cos(\alpha+\beta) \cdot \Delta\alpha_i)) \cdot (b \cdot \sin\alpha - q \cdot \sin(\alpha+\beta))}. \tag{27}$$

Similarly, we obtain the expression for Δp_{β_i}:

$$\Delta p_{\beta_i} = \frac{\Delta\beta_i \cdot a \cdot q \cdot \sin\alpha \cdot (b \cdot \cos\beta - q)}{(b \cdot \sin\alpha - q \cdot \sin(\alpha+\beta))^2 - \Delta\beta_i \cdot q \cdot \cos(\alpha+\beta) \cdot (b \cdot \sin\alpha - q \cdot \sin(\alpha+\beta))}. \tag{28}$$

Using expressions (18) and their corresponding (8), (9) and (7), as well as (23)–(26), we obtain expressions for determining the error bounds of $\Delta p_{\alpha'_i}, \Delta p_{\alpha''_i}$:

$$\Delta p_{\alpha'_i} = \frac{a \cdot q \cdot \Delta O \cdot \sin\beta}{(b \cdot (a \cdot \sin\alpha + \Delta O \cdot \cos\alpha) - q \cdot (a \cdot \sin(\alpha+\beta) + \Delta O \cdot \cos(\alpha+\beta)))} \cdot \frac{(q \cdot \cos(\alpha+\beta) - b \cdot \cos\alpha)}{(b \cdot \sin\alpha - q \cdot \sin(\alpha+\beta))}, \tag{29}$$

$$\Delta p_{\alpha_i''} = \frac{q\cdot a\cdot \Delta A\cdot \sin^2\beta}{\left(b\cdot\left(\sin\alpha+\cos\alpha\cdot\frac{\Delta A\cdot\sin\beta}{a+b+\Delta\ \ \cdot\cos\beta}\right)-q\cdot\left(\sin(\alpha+\beta)+\cos(\alpha+\beta)\cdot\frac{\Delta A\cdot\sin\beta}{a+b+\Delta A\cdot\cos\beta}\right)\right)}\cdot\frac{(q\cdot\cos(\alpha+\beta)-b\cdot\cos\alpha)}{(b\cdot\sin\alpha-q\cdot\sin(\alpha+\beta))\cdot(a+b+\Delta A\cdot\cos\beta)}. \quad (30)$$

The error limits δp_{a_i}, δp_{b_i}, δp_{α_i}, δp_{β_i}, $\delta p_{\alpha_i'}$, $\delta p_{\alpha''_i}$ in relative form will be determined according to the expressions:

$$\delta p_{a_i} = \frac{\Delta p_{a_i}}{p},\ \delta p_{b_i} = \frac{\Delta p_{b_i}}{p},\ \delta p_{\alpha_i'} = \frac{\Delta p_{\alpha_i'}}{p},\ \delta p_{\alpha_i''} = \frac{\Delta p_{\alpha_i''}}{p},\ \delta p_{\alpha_i} = \frac{\Delta p_{\alpha_i}}{p},\ \delta p_{\beta_i} = \frac{\Delta p_{\beta_i}}{p}. \quad (31)$$

After performing the algebraic transformations of expressions (31), (20), (21), (27)–(30), we have:

$$\delta p_{a_i} = \frac{\Delta a_i}{a}, \quad (32)$$

$$\delta p_{b_i} = \frac{-\Delta b_i\cdot\sin\alpha}{\sin\alpha\left(b+\Delta b_i\right)-q\cdot\sin(\alpha+\beta)}, \quad (33)$$

$$\delta p_{\alpha_i} = \frac{\Delta\alpha_i\cdot(q\cdot\cos(\alpha+\beta)-b\cdot\cos\alpha)}{b\cdot\sin\alpha-q\cdot\sin(\alpha+\beta)-\Delta\alpha_i\cdot(q\cdot\cos(\alpha+\beta)-b\cdot\cos\alpha)}, \quad (34)$$

$$\delta p_{\beta_i} = \frac{\Delta\beta_i\cdot\sin\alpha\cdot(b\cdot\cos\beta-\ \ )}{\sin\beta\cdot\left((b\cdot\sin\alpha-\ \ \cdot\sin(\alpha+\beta))-\Delta\beta_i\cdot q\cdot\cos(\alpha+\beta)\right)}, \quad (35)$$

$$\delta p_{\alpha_i'} = \frac{\Delta O\cdot(q\cdot\cos(\alpha+\beta)-b\cdot\cos\alpha)}{b\cdot(a\cdot\sin\alpha+\Delta O\cdot\cos\alpha)-q\cdot(a\cdot\sin(\alpha+\beta)+\Delta O\cdot\cos(\alpha+\beta))}, \quad (36)$$

$$\delta p_{\alpha_i''} = \frac{\Delta A\cdot\sin\beta}{\left(b\cdot\left(\sin\alpha+\cos\alpha\cdot\frac{\Delta A\cdot\sin\beta}{a+b+\Delta A\cdot\cos\beta}\right)-q\cdot\left(\sin(\alpha+\beta)+\cos(\alpha+\beta)\cdot\frac{\Delta A\cdot\sin\beta}{a+b+\Delta\ \ \cdot\cos\beta}\right)\right)}\cdot\frac{(q\cdot\cos(\alpha+\beta)-b\cdot\cos\alpha)}{(a+b+\Delta A\cdot\cos\beta)}. \quad (37)$$

Technological deviations of parameters and their respective error components can be reasonably considered as uncorrelated (or weakly correlated) in the general case [15]. However, in real conditions, when manufacturing, assembling, and adjusting a laser triangulation sensor, it is most economically feasible to use only one sample of a linear dimension measuring tool and one sample of an angular dimension measuring tool. This factor can lead to correlations between the respective error components. Therefore, to combine the components of technological error, we will take into account the correlation between $\delta p_{a_i}, \delta p_{b_i}, \delta p_{\alpha_i}, \delta p_{\beta_i}$, and $\delta p_{\alpha_i}, \delta p_{\beta_i}$. The statistical confidence bounds of the total systematic relative error $\delta_{i\Sigma}$ are found by the expression:

$$\delta p_{i\Sigma} = \sqrt{\sum_{j=1}^{m}\delta_j^2} = \sqrt{\left(\delta p_{a_i}+\delta p_{b_i}+\delta p_{\alpha_i'}+\delta p_{\alpha_i''}\right)^2+\left(\delta p_{\alpha_i}+\delta p_{\beta_i}\right)^2} \quad (38)$$

To determine the statistical confidence limits of the total systematic absolute error, we use the expression

$$\Delta p_{i\Sigma} = \delta p_{i\Sigma}\cdot p, \quad (39)$$

where p is the measurement range.

Thus, the set of expressions (20), (21), (27)–(30), (32)–(37) for estimating the statistical confidence limits of the total systematic error caused by technological deviations of the parameters of a laser triangulation sensor is a mathematical model of the technological error of a laser triangulation sensor.

We will quantify the components of the technological error of a laser triangulation sensor for the case of using common means of measuring linear dimensions and angular values, which are mass-produced by domestic and foreign manufacturers, for example, the TESA MICRO-HITE 3D coordinate measuring machine [16] and the GS-1L goniometer [17].

The combination of parameters of the optical scheme of the laser triangulation sensor will be chosen to correspond to the characteristics of the main types of weapons in service with the National Guard of Ukraine, which, according to the calibre ranges, belong to small arms (7,62 mm – 19,99 mm), artillery of small (20,00 mm – 75,99 mm), medium (76,00 mm – 151,99 mm) and large (152,00 mm and more) calibres [18].

The results of the calculations are presented in the form of tables and graphs of the dependences of the limits of relative $\delta p_{i\Sigma}(p)$ and absolute $\Delta p_{i\Sigma}(p)$ technological errors on the increase in the radius of bore of the measurement range. Examples of dependencies $\Delta p_{i\Sigma}(p)$ are shown in Figures 4–7.

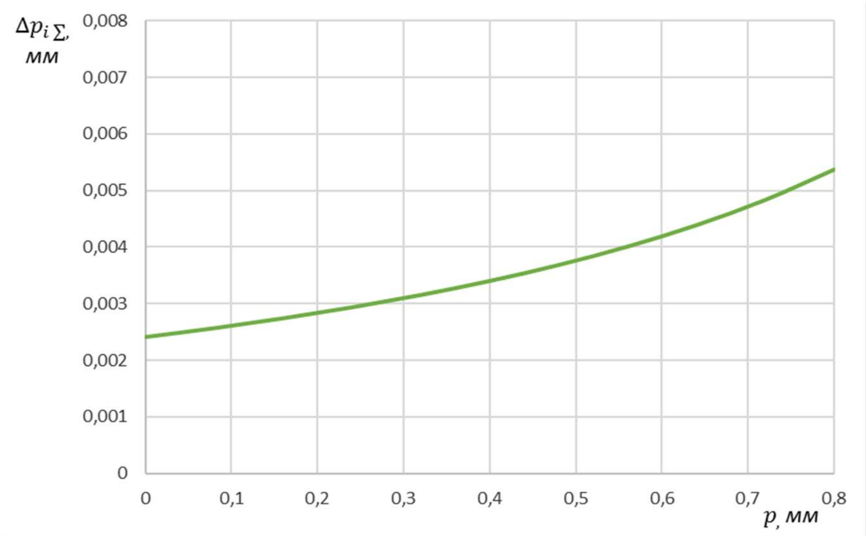

Figure 4 – Dependence graph $\Delta p_{i\Sigma}(p)$ for a laser triangulation sensor, corresponding to the parameters of the small arms bore

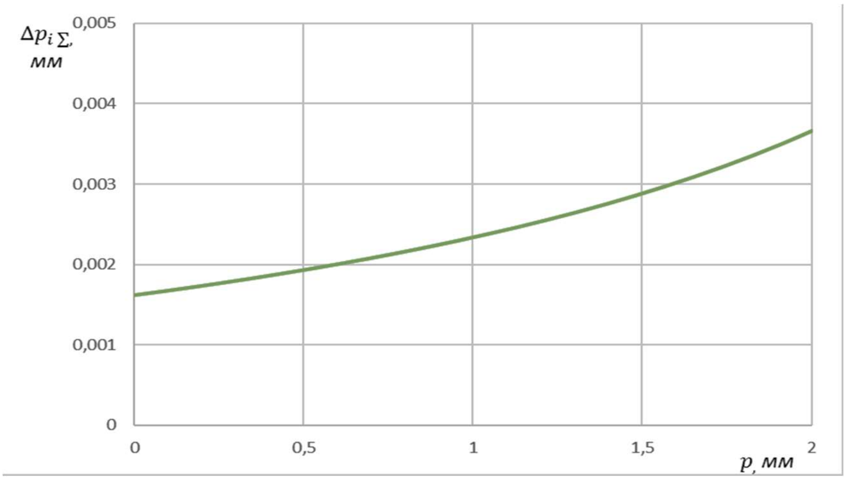

Figure 5 – Dependence graph $\Delta p_{i\Sigma}(p)$ for a laser triangulation sensor, corresponding to the parameters of the small-calibre artillery bore

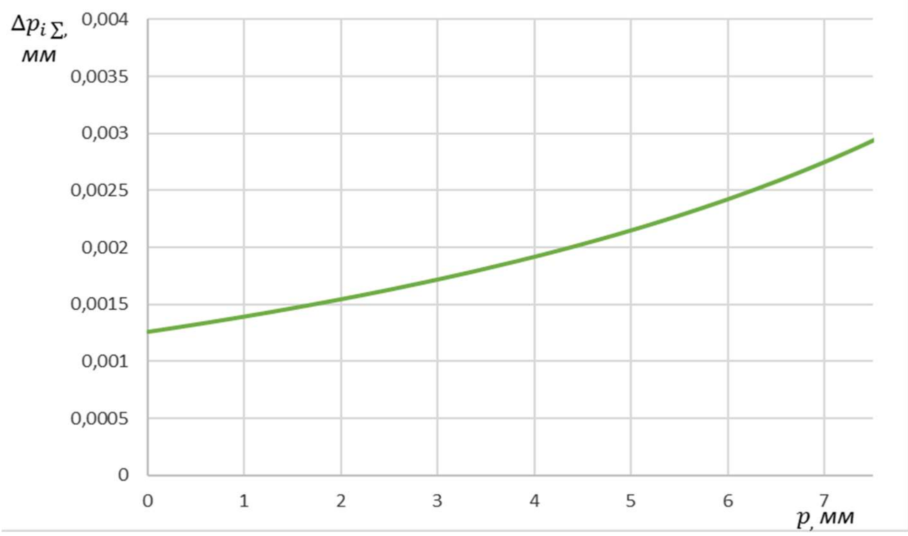

Figure 6 – Dependence graph $\Delta p_{i\Sigma}(p)$ for a laser triangulation sensor, corresponding to the parameters of the medium-calibre artillery bore

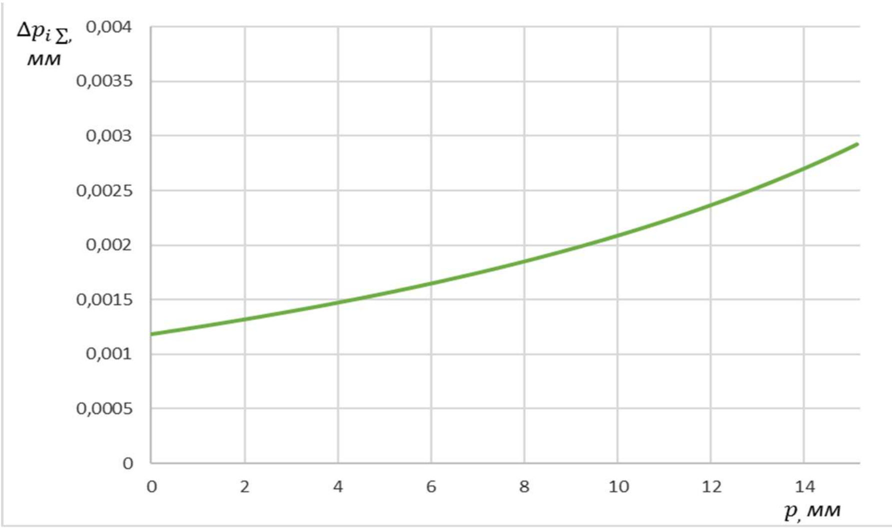

Figure 7 – Dependence graph $\Delta p_{i\Sigma}(p)$ for a laser triangulation sensor, corresponding to the parameters of the bore of large-calibre artillery

The analysis of the graphs shows that the technological error increases with the approach to the upper limit of the measurement range and is minimal at its beginning. Such a property of the error is quite acceptable from the point of view of the measurement concept, according to which the greatest value of information about the parameters of the bore is seen in the initial part of the measurement range (i.e., the one corresponding to the least wear of the bore and the initial stage of its degradation).

In addition, a decrease in the technological error is observed with an increase in the barrel calibre. This fact can be explained by the fact that in the case of increasing the diameter of the bore, the space for the location of the elements of the optical scheme of the laser triangulation sensor increases and the distances *a* (from the intersection of the main optical axes of the focusing and receiving lenses to the optical centre of the receiving lens) and *b* (from the optical centre of the receiving lens to the surface of the photosensitive detector) increase. Taking into account the peculiarities of expressions (32)–(37), in which the parameters a and b are included as arguments, an increase in the latter leads to a decrease in the value of the technological error limit.

Conclusions

Analysis of the results of quantitative evaluation shows that the limit of absolute technological error is within ± 0,0012 mm – 0,0050 mm. These values allow us to preliminarily assume that the method and measuring instrument discussed in this article have a certain margin of safety with respect to the requirements for the accuracy of measurements of the bore parameters.

The final conclusion on the limits of applicability of the laser triangulation tool for measuring the geometric characteristics of the bores can be made after research aimed at modelling and quantifying the impact of temperature deviations and other factors on the measurement results.

Further areas of research should include the development of mathematical models of other components of the total error of the measuring instrument and the justification of the method of their calculation combination.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТА АНАЛІЗ ТЕХНОЛОГІЧНОЇ ПОХИБКИ ЗАСОБУ ВИМІРЮВАННЯ ГЕОМЕТРИЧНИХ ХАРАКТЕРИСТИК КАНАЛІВ СТВОЛІВ ВОГНЕПАЛЬНОЇ ЗБРОЇ

Обґрунтовано реалізацію контролю геометричних характеристик каналів стволів вогнепальної зброї, отримано математичні моделі складових технологічної похибки, а також проведено кількісне оцінювання діапазонів варіювання границь технологічної похибки для характерних сполучень умов виконання вимірювань геометричних характеристик каналів стволів вогнепальної зброї.

Проведено аналіз публікацій стосовно відомих методів вимірювання геометричних характеристик каналів стволів із застосуванням лазерного тріангуляційного датчика, підходів до математичного моделювання складових похибки чутливих елементів прецизійних засобів вимірювань. Установлено, що попри значну кількість публікацій (матеріалу), у них не ставилися і не розв'язувалися завдання математичного моделювання похибки лазерного тріангуляційного датчика, зумовленої недосконалістю технології процесів виготовлення, збирання та юстирування засобу вимірювання.

Визначено параметри оптичної схеми лазерного тріангуляційного датчика, відхилення яких від їхніх номінальних (розрахункових) значень чинитиме вплив на функцію перетворення. Прояви технологічних відхилень кожного з параметрів оптичної схеми було розглянуто через їх вплив на функцію перетворення лазерного тріангуляційного датчика окремо та незалежно одне від одного і розроблено математичні моделі для визначення впливу відхилень параметрів оптичної схеми, які сприймаються вимірювальним каналом засобу вимірювання як поява позірних приростів вимірювальної величини. Установлено довірчі границі сумарної технологічної похибки із врахуванням кореляції між результатами вимірювань різних параметрів одним і тим самим засобом вимірювання. Проведено кількісне оцінювання технологічної похибки для різних характерних сполучень параметрів зброї із визначенням границь допустимої абсолютної технологічної похибки для чотирьох діапазонів калібрів.

У результаті розрахунків отримано дані, які подано у вигляді графіків залежностей границь абсолютної технологічної похибки від приросту радіуса каналу ствола у діапазоні вимірювань. Проведено аналіз одержаних даних, який дає змогу попередньо припустити, що метод і засіб вимірювання, які розглядаються у статті, мають певний запас відносно вимог до точності вимірювань параметрів каналів стволів.

Отже, запропоновано математичну модель складових технологічної похибки, а також виконано кількісне оцінювання діапазонів варіювання границь технологічної похибки для характерних сполучень умов виконання вимірювань геометричних характеристик каналів стволів вогнепальної зброї.

Ключові слова: *математична модель, канал ствола, технічний стан, метод діагностування, лазерний тріангуляційний датчик, геометрична характеристика, технологічна похибка, засіб вимірювання, технічне діагностування, складова похибки.*

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