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MODELING AND FORECAST THE RESULTS OF ANTI-AIRCRAFT MISSILE GROUPING'S AIR DEFENSE BATTLE

Using a real example and a special method, the essential properties and internal law of anti-aircraft missile grouping's air defense battle were revealed. A statistically sound mathematical modeling apparatus in the class of Markov processes with continuous time and discrete states was selected. A model with an analytical description of battle internal law was developed, its adequacy to real battle was tested, and an estimate of predicting battle results accuracy was found.

Keywords: *air defense, anti-aircraft missile forces grouping, Markov processes, modeling.*

Statement of the problem. The task of protecting the troop's groupings and the state important objects from air strikes in course of hostilities is assigned to the anti-aircraft missile forces (AAMF) as part of air-defense (AD) missile brigades, regiments and subunits, which are deployed in advance in battle formations in positional areas and form an anti-aircraft missile (AAM) grouping.

The AAM grouping performs the air defense battle mission during an air-defense battle as a set of consecutive and simultaneous air-defense battles (fire contacts) of separate surface-to-air missile divisions (SAMDs) with means of enemy air attack (MAA). The participants' composition, the number and parameters of fire contacts in space and in time of grouping's battle, as well as possible losses of sides are not known in advance (are random).

However, each time when building the combat order of the AAM grouping, the practical task of the grouping's air-defense battle predicting results arises, namely the losses of enemy's MAA and grouping's surface-to-air missile systems (SAMS), the number of fire contacts, the sufficiency of anti-aircraft guided missiles (AAGM) stock and the composition of the surface-to-air missile systems in grouping to repel the first and subsequent strikes of enemy's MAA.

As a result, the current problem of finding a stable internal law of air-defense battle, and on its basis, building a model for predicting the values of the above mentioned indicators arises.

Analysis of recent research and publications. Based on the composition of the mathematical tools used, the areas of known work on modeling the actions of anti-aircraft missile systems AD and missile defense can be conditionally divided into several categories. Thus, when constructing models for assessing the effectiveness of air defense and missile defense of ground objects [1–4] and a grouping of surface ships [5], the mathematical apparatus of queuing theory [1, 5], game theory [2], Petri nets [4], as well as the idea of heterogeneous networks [3] were used. At the same time, the authors intuitively believe that the chosen mathematical apparatus corresponds to the processes of air defense battle of AAM grouping. In favor of this conclusion, the results of simulation modeling are presented [2, 3, 4].

However, the listed mathematical methods do not take into account the main factor of air defense battle – the possibility of SAM systems damage, which makes the noted models devoid of adequacy to real combat processes.

The purpose of the article is to identify the internal law of the anti-aircraft missile forces grouping air-defense battle and, on its basis, to develop and assess the adequacy of the model as a tool for predicting the values of the battle's above mentioned indicators results.

Summary of the main material. To achieve the goal, we will use elements of a special technology for developing and guaranteed production of models [7] and a description of a real air-defense battle on June 30, 1970, involving an AAM grouping consisting of three SAMS SA-3 and thirteen SAMS SA-2, which was reinforced by portable surface-to-air missile systems (PSAMS) units in the Suez Canal zone (Figure 1, Table 1)

If we briefly trace the battle development then we can see, that the first group of MAA, consisting of 4 aircraft (2 – Skyhawk and 2 – Phantom), at an altitude of 50 meters, under the cover of the terrain, entered the grouping's deployment area. One aircraft was shot down by a portable SAM system, the second aircraft was unsuccessfully shot upon by SAMS No. 12 SA-3 type. The non-hit aircraft of the first group attacked and hit the SA-2 SAMS No. 13 and left the zone of fire of AAM grouping. At this time, the subunits of the AAM grouping were put on alert. Therefore, the second group of aircraft, which entered the fire zone of grouping on the opposite flank and at an altitude of 400 meters, was met by fire from two SA-2 SAMS (No. 3 and No. 7). One of the attacking planes was shot down by SA-2 SAMS No. 7. Its partner refused to carry out the combat mission, turned around and quickly left the grouping's zone of fire. The remaining two aircraft were unsuccessfully shot upon by SA-2 SAMS No. 4 and SA-2 SAMS No. 1, continued flying, attacked and hit SA-2 SAMS No. 3, and then they were unsuccessfully shot upon by SA-2 SAMS No. 2 and quickly left the groupings fire zone.

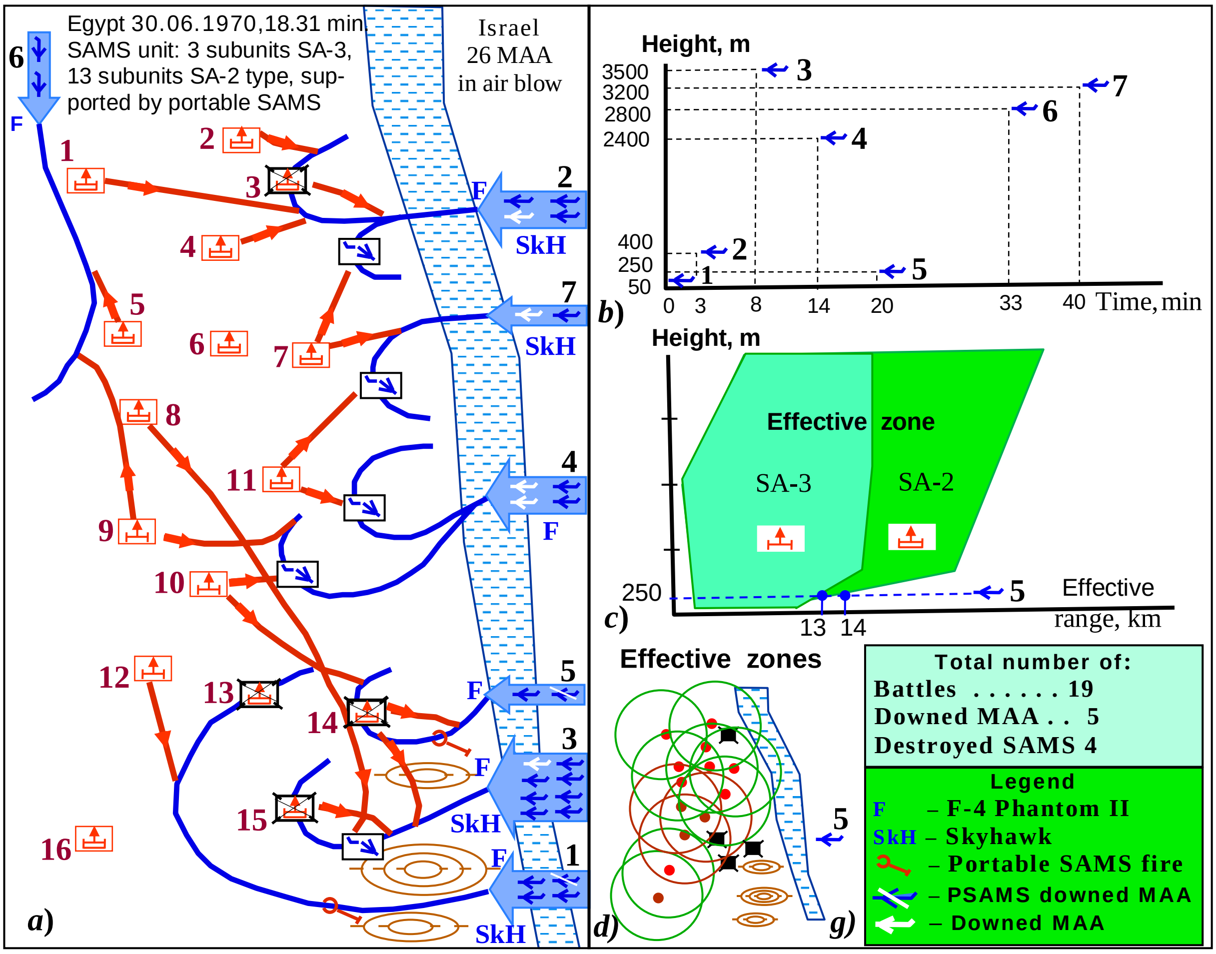

Figure 1– Scheme and parameters of a real air defense battle on 30.06.1970 in the Suez Canal zone: ***a***) a map-diagram of the battle dynamics; ***b***) the MAA strike's height-time diagram; ***c***) the SAM affective areas structure; ***d***) total affective area of SAM unit at MAA No. 5 flight altitude; ***g***) final battle results and designations on the map-diagram

Table 1 – Estimates of current and integral parameters* of the battle (*see Figure* 1)

No.	The SAMS unit air defense battle events time, minutes			Battle participants' numbers		Losses in the battle contact	No.	Integral parameters of SAMS unit's air defense battle	
	Start (&MAA)	End	Δt_{SAMS}	SAMS	MAA				
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0.50	1.23	0.73	PSAMS	1	1 aircraft	25	n_{fc}	22
2	2.02	2.93	0.91	12	1	–	26	$n_{d\ SAMS}$	4
3	3.28	3.63	0.35	13	1	SAMS No. 13	27	N_{sda}	5
4	3.02	3.70	0.68	3	2	–	28	P^*	0.181818
5	2.96	4.03	1.07	7	2	1 aircraft	29	P_{sda}	0.227273
6	3.47	4.13	0.66	4	2	–	30	n_0	16
7	2.40	4.30	1.90	1	2	–	31	$D(n_{fc})$	0.011364
8	3.81	4.43	0.62	3	2	SAMS No. 3	32	$D(n_{d\ SAMS})$	0.062500
9	4.53	4.93	0.40	2	2	–	33	$D(N_{sda})$	0.058000
10	7.74	8.63	0.89	14	3	–	34	$n_{fc\ \infty}$	88
11	8.42	8.73	0.31	15	3	–	35	$N_{sda\ \infty}$	20
12	6.84	9.10	2.26	8	3	1 aircraft	Estimates with inner law		
13	8.82	9.43	0.61	15	3	SAMS No. 15	36	w (44)	0.251485
14	14.26	15.11	0.85	11	4	1 aircraft	37	n_{fc}	22.130652
15	15.23	15.80	0.57	10	4	1 aircraft	38	$n_{d.S}$	4.023755
16	14.70	16.23	1.53	9	4	–	39	N_{sda}	5.029694
17	20.08	20.40	0.32	14	5	–	40	Δ, %	0.593873
18	20.60	20.60	0.25	PSAMS	5	1 aircraft	41	$\chi^2_{Exp.SAMS}$	3.945236
19	20.63	21.00	0.37	14	5	SAMS No. 14	42	$\chi^2_{Norm.SAMS}$	7.597282
20	19.66	21.53	1.87	10	5	–	43*	$\chi^2_{Exp.MAA}$	1.675774
21	34.08	34.47	0.39	5	6	–	44*	$\chi^2_{Norm.MAA}$	32.963587
22	33.46	35.10	1.64	9	6	–	45*	α level	0.050000
23	40.25	40.50	0.25	7	7	–	46*	$\chi^2_{Critical}$ (r =1, α)	3.890000
24	39.92	41.10	1.18	11	7	1 aircraft	47*	$\chi^2_{Critical}$ (r =2, α)	5.990000

****Note.*** $\chi^2_{Norm.SAMS} = 7.597 > 3.89$; $\chi^2_{Norm.MAA} = 32.96 > 3.89$; $a_\Delta = (0.02; 0.263; 0.4; 1.08; 6.83)$;
$\chi^2_{Exp.SAMS} = 3.945 < 5.99$; $\chi^2_{Exp.MAA} = 1.675 < 5.99$; $b_\Delta = (0.25; 0.53; 0.84; 1.34; 2.26)$.

The battle further development can be traced according to the map-scheme (Fig. 1***a***), according to the height-time diagram (Fig. 1***b***) and according to Table 1. Every time, depending on the altitude of aircraft flight, the fire zone range of each SAM systems was changed in accordance with its characteristics (Fig. 1***c***), which led to changes in the grouping's effective zones coefficient of overlapping (Fig. 1***d***) and to changes in the degree of aircraft accessibility to be shot upon by SAMS of AAM grouping.

The fight lasted 44 minutes. In the result of the battle (Fig. 1***g***), subunits of AAM grouping made 22 fire contacts. Out of the 26 enemy aircraft, 5 aircraft were shot down by SA-2 and SA-3 type SAMS subunits and two aircraft were shot down by portable SAMS. Air enemy managed to hit 4 SAM systems.

This description (Figure 1 and Table 1) makes it possible to select the following significant features of the AAM grouping air defense battle.

1. Air attack means operated in groups with a composition that was not known in advance (random for the AAM grouping), with a location and time of entry into the fire zone of individual SAM systems unknown in advance.

2. The range of each SAMS' fire zone was determined by its characteristics (Fig. 1***c***) and changed each time depending on the MAA flight altitude, which led to unpredictable changes in grouping total zone of fire (Fig. 1***d***) and its overlap coefficient.

3. Enemy aircraft, at best, carried out one attack of SAMS and did not remain in the AAM grouping's firing zone for repeated battles.

The main recurring element of AAM grouping anti-aircraft combat is air-defense battle (fire contact) of a single SAMS with enemy MAA. The starting, ending, and results of each fire contact are not known in advance – they are random. In this case, the results of each fire contact can be: destruction of the enemy's MAA (with probability P_{sda}); non-destruction of the enemy's MAA (with probability $1-P_{sda}$); defeat of the SAMS (with probability P^*); non-defeat of the SAMS (with probability $1-P^*$); combinations of the above mentioned results.

When searching for the main parameters that determine the development of AAM grouping's anti-aircraft combat it can be argued that such parameters are the random time intervals between the start of individual SAMS fire contacts and the duration of such contacts.

Let's formulate the main hypotheses about the essential properties of AAM grouping's anti-aircraft combat:

1) the result of each SAMS' fire contact is random – it is not known in advance and can include both the destruction of the enemy's MAA and the defeat of SAMS;

2) each SAMS fire contact develops over time as a random process, the beginning and end times of which are not known in advance (are random).

To find the internal law of AAM grouping's anti-aircraft combat, we will use the above mentioned hypotheses and find the mathematical expectation [6] of enemy MAAs shot down number $N_{sda.1}$ and SAMS defeat number $n_{d.S.1}$ during one fire contact:

$$N_{sda.1} = 1\cdot P_{sda} + 0\cdot(1-P_{sda}) = P_{sda},$$
$$n_{d.S.1} = 1\cdot P^* + 0\cdot(1-P^*) = P^*. \quad (1)$$

Let us assume that at the time t of the battle the average number of SAMS fire contacts was n_{fc}. For convenience, we use the full and simplified notation of mathematical expectation N_{sda} of enemy MAAs (aircraft) shot down number at the time t as $M[N_{sda}] = N_{sda}$ and SAMS defeated $n_{d.S}$ as $M[n_{d.S}] = n_{d.S}$, we obtain:

$$M[N_{sda}] = M\left[\sum_{i=1}^{n_{fc}} N_{sda.1}\right] = \sum_{i=1}^{n_{fc}} M[N_{sda.1}] = \sum_{i=1}^{n_{fc}} P_{sda} = P_{sda}\cdot n_{fc}, \text{ then } N_{sda} = P_{sda}\cdot n_{fc}; \quad (2)$$

$$M[n_{d.S}] = M\left[\sum_{i=1}^{n_{fc}} n_{d.S.1}\right] = \sum_{i=1}^{n_{fc}} M[n_{d.S.1}] = \sum_{i=1}^{n_{fc}} P^* = P^*\cdot n_{fc}, \text{ then } n_{d.S} = P^*\cdot n_{fc}. \quad (3)$$

Next, we will take into account the limited number of SAMS in the AAM grouping and the possibility of each SAMS being defeated by enemy MAA fire in each fire contact. For conditions of unlimited supply of AAGM in each SAMS and unlimited number of enemy MAAs entering the battle in turn, we direct the battle time to infinity (Figure 2).

In this case, all n_0 SAMS in the AAM grouping will be eventually defeated. At this point, the number of fire contacts will reach a limit value $n_{fc\infty}$:

$$\lim_{t\to\infty} n_{d.S}(t) = n_0, \quad \lim_{t\to\infty} n_{fc}(t) = n_{fc\infty}. \quad (4)$$

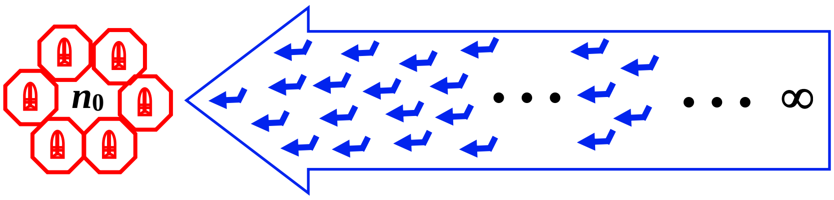

Figure 2 – Boundary conditions of anti-aircraft combat

Let us substitute the limiting values $n_{\text{d.S}}$ and $n_{\text{fc}\infty}$ from equation (4) into the right-hand side of equation (3) and find the estimate the mathematical expectation of maximum number of SAMS fire contacts $n_{\text{fc}\infty}$ with the enemy's MAA until the moment of all n_0 SAMS in the AAM grouping are defeated, as well as the maximum number of fire contacts for one SAMS:

$$n_0 = P^* \cdot n_{\text{fc}\infty}, \text{ then } n_{\text{fc}\infty} = \frac{n_0}{P^*}$$

$$\text{and } n_{\text{fc}\infty}(n_0 = 1) = \frac{1}{P^*}. \qquad (5)$$

It is not difficult to verify the correctness of equation (5) physical content. So if the probability of SAMS defeat during one fire contact is equal to unity, i.e. $P^* = 1$, then the mathematical expectation of fire contacts number will coincide with the number n_0 of SAMS in the AAM grouping.

By substituting the value $n_{\text{fc}\infty}$ from equation (5) into equation (2), we find the limiting value $N_{\text{sda}\infty}$ of enemy MAAs shot down mathematical expectation number until the moment when all SAMS in the AAM grouping are defeated:

$$N_{\text{sda}\infty} = P_{\text{sda}} \cdot n_{\text{fc}\infty} = n_0 \frac{P_{\text{sda}}}{P^*}. \qquad (6)$$

Let's find the relative values of number of fire contacts $n^*_{\text{fc}}(t)$, downed enemy MAAs $N^*_{\text{sda}}(t)$ and SAMS losses $n^*(t)$:

$$n^*_{\text{fc}}(t) = \frac{n_{\text{fc}}(t)}{n_{\text{fc}\infty}}; \qquad (7)$$

$$N^*_{\text{sda}}(t) = \frac{N_{\text{sda}}(t)}{N_{\text{sda}\infty}}; \qquad (8)$$

$$n^*(t) = \frac{n_{\text{d.S}}(t)}{n_0}. \qquad (9)$$

Let us divide the left and right sides of equation (3) by the number n_0 of SAMS in the AAM grouping and take into account equalities (5) and (9); we will obtain the equality of defeated $n^*(t)$ SAMS and fire contacts $n^*_{\text{fc}}(t)$ mathematical expectations relative values number at any moment of AAM grouping battle time:

$$\frac{n_{\text{d.S}}(t)}{n_0} = \frac{P^* \cdot n_{\text{fc}}(t)}{n_0} = \frac{n_{\text{fc}}(t)}{n_0 / P^*},$$

$$\text{then} \qquad n^*(t) = \frac{n_{\text{fc}}(t)}{n_{\text{fc}\infty}} = n^*_{\text{fc}}(t). \qquad (10)$$

Let's multiply the numerator and denominator of the equation (10) right side by the probability P_{sda} of MAA shoot down as a result of a single fire contact:

$$n^*(t) = \frac{P_{\text{sda}} \cdot n_{\text{fc}}(t)}{P_{\text{sda}} \cdot n_{\text{fc}\infty}} = \frac{N_{\text{sda}}(t)}{N_{\text{sda}\infty}} = N^*_{\text{sda}}(t). \qquad (11)$$

Equations (10) and (11) were obtained using only admissible operations with mathematical expectations of random variables and based on the most significant hypotheses 1 and 2 about the AAM grouping's anti-air combat processes.

This allows us to formulate an internal law of anti-aircraft combat: the relative values of the mathematical expectations of the AAM grouping's SAMS fire contacts number and the losses of sides are equal to each other at any moment in time of the battle:

$$n^*(t) = N^*_{\text{sda}}(t) = n^*_{\text{fc}}(t) = w(t). \qquad (12)$$

At the same time, the absolute values of the above mentioned parameters may have different values depending on the battle conditions. If, during the development of the battle model analytical description, equation (12) for the relative losses of the sides is found, then such a model can be considered adequate for a real battle of AAM grouping with the accuracy of hypotheses 1 and 2 about the most essential properties of the battle processes.

Let us return to the example of combat (Figure 1, Table 1) and note that at the moment of each end of fire contact, the normalized values of variables (7), (8), (9) may increase (Figure 3) by the corresponding value (Table 1, items 31, 32, 33):

$$D(n^*_{\text{fc}}) = \frac{1}{n_{\text{fc}\,\infty}} = \frac{P^*}{n_0}, \qquad D(n^*) = \frac{1}{n_0},$$

$$D(N^*_{\text{sda}}) = \frac{1}{N_{\text{sda}\infty}} = \frac{P^*}{n_0 P_{\text{sda}}}. \qquad (13)$$

Let us apply the least squares method to the considered example (Figure 1) and obtain the smoothed lines of these dependence quantities and magnitude of the sides relative losses $w(t)$ on time (Figure 3):

$$\left.\begin{aligned} n^*_{\text{fc}}(t) &= 0.0831 \cdot \ln(t) - 0.0703 \qquad N^*_{\text{sda}}(t) = 0{,}0886 \cdot \ln(t) - 0.0976 \\ n^*(t) &= 0.088 \cdot \ln(t) - 0.0604 \qquad w(t) = 0.08567 \cdot \ln(t) - 0.0761 \end{aligned}\right\}. \qquad (14)$$

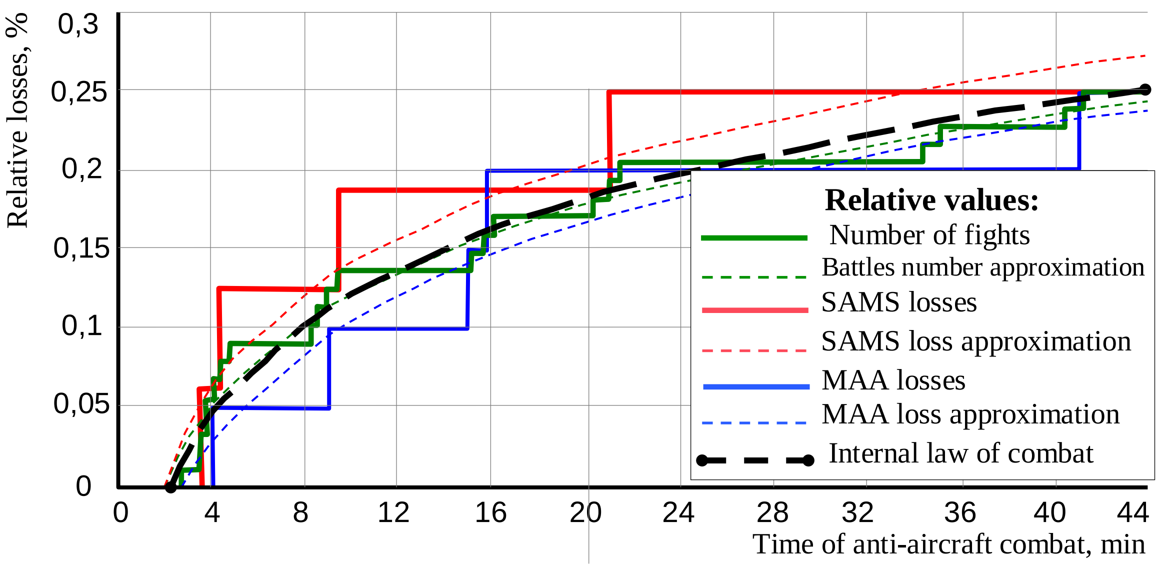

Figure 3 – Dependence of relative sides' losses on the time of battle

Using expressions (5), (7), (8) and (9), it is possible to find estimates of mathematical expectations absolute values for fire contacts and losses of parties:

$$n_{\text{fc}}(t) = w(t) \cdot n_{\text{fc}\infty},$$

$$N_{\text{sda}}(t) = w(t) \cdot N_{\text{sda}\,\infty},$$

$$n_{\text{d.S}}(t) = w(t) \cdot n_0. \qquad (15)$$

At the time ($t = 44$ min) of the MAA blow repelling end, one can find the values of sides relative $w(44)$ and absolute losses (Table 1, items 36–39), which turn out to be overestimated by $\Delta = 0{,}59$ % (Table 1, item 40). However, the fact of a relatively accurate coincidence of the obtained estimates with the real results testifies in favor of correctness the found internal law (12) in anti-aircraft combat of AAM grouping.

For the practical application of the above law (12), we will develop a model of combat and find a variant of its analytical description.

The adequacy of such a model is possible only when choosing a mathematical apparatus that corresponds to the distribution laws of the main random parameters [7] that determine the development of battle: time intervals between the entry of MAAs into the fire zone (Table 1, column 2, $\Delta t_{\text{MAA}.i} = t_{\text{start}.i+1} - t_{\text{start}.i}$); duration of the SAMS firing cycle (Table 1, column 4, $\Delta t_{\text{SAMS}.i} = t_{\text{end}.i} - t_{\text{start}.i}$).

Under the conditions of the battle, the normal and exponential distribution laws are competing. The results of their evaluation in each sample of random variables with interval boundaries a_Δ for the MAA strike and b_Δ for the SAMS battle firing cycles and with the corresponding number of freedom degrees r, according to the Pearson criterion (Table 1, items 41–47; note – criterion χ^2) with a significance level $\alpha_{\text{level}} = 0{,}05$, determine the need to reject the hypothesis of their normal distribution and accept the hypothesis of their exponential integral distribution law. As a result of the combat model (sequence of fire contacts), the AAM grouping should be built in the class of Markov processes with continuous time and with discrete states.

In the first step, given the limited allowable scope of the article, we will build a model for the key element of MAA strike repelling – for the sequence of a single-channel SAMS fire contacts against a flow of single MAAs with intensity of I MAAs per minute.

We will denote the possible states of SAMS in battle by the symbol S_{ij}, where the first index i will be used to indicate the number of SAMS defeated in this state; the second index j will be used to indicate the number of enemy MAAs that are being fired on in this state.

We obtain a graph of the anti-aircraft battle model of a separate SAMS (Figure 4), where the transition from state S_{00} to state S_{01} is possible in case of the next enemy MAA detection and is characterized by the intensity (frequency) of fire contacts I.

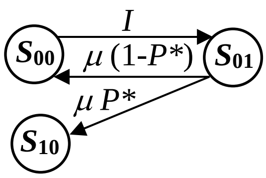

Figure 4 – Graph of the surface-to-air missile system's anti-aircraft combat simplest model

Each fire contact can last a random time T_{random}, which has an exponential distribution law with a mathematical expectation T_{avr}, with a parameter μ and with the intensity I of fire contacts occurrence:

$$M[T_{\text{random}}] = T_{\text{avr}},$$

$$\mu = \frac{1}{T_{\text{avr}}}, \qquad I = \frac{N_{\text{MAA}}}{T_{\text{str}}}. \qquad (16)$$

Each fire contact can result in SAMS being defeated with probability P^* and the process transition (Figure 4) from state S_{01} to state S_{10} or with probability $(1-P^*)$ to have a successful outcome for SAMS, causing a transition from state S_{01} to state S_{00}.

The exponential distribution of random variables in the battle process makes it possible to construct a system of Kolmogorov differential equations [8] for the probabilities of battle model states S_{ij} (Figure 4), where for convenience we will denote the derivatives of states' probabilities P_{ij} by a point and we will not mark the dependence of the probabilities on time in formula (17):

$$\begin{aligned} \dot{P}_{00} &= -I \cdot P_{00} + \mu \cdot (1-P^*) \cdot P_{01}, \\ \dot{P}_{01} &= -\mu \cdot P_{01} + I \cdot P_{00}, \\ \dot{P}_{10} &= \mu \cdot P^* \cdot P_{01}. \end{aligned} \qquad (17)$$

Let us integrate the system of equations (17) under the initial conditions:

$$P_{00}(t=0) = 1; \; P_{01}(t=0) = P_{10}(t=0) = 0. \qquad (18)$$

We will get:

$$\begin{aligned} P_{00} &= C_1 \cdot e^{\lambda_1 \cdot \mu t} + C_2 \cdot e^{\lambda_2 \cdot \mu t}, \\ P_{01} &= \frac{\rho}{\alpha}\left[e^{\lambda_1 \cdot \mu t} - e^{\lambda_2 \cdot \mu t} \right], \\ P_{10} &= 1 - \frac{1}{\alpha}\left[\lambda_1 \cdot e^{\lambda_2 \cdot \mu t} - \lambda_2 \cdot e^{\lambda_1 \cdot \mu t} \right], \end{aligned} \qquad (19)$$

where

$$\rho = \frac{I}{\mu}, \quad \alpha = \sqrt{1+\rho^2+2\rho\left(1-2P^*\right)},$$

$$C_{12} = \frac{\alpha \pm (1-\rho)}{2\alpha}, \quad \lambda_{12} = \frac{\pm\alpha - (1+\rho)}{2}. \qquad (20)$$

The mathematical expectation of the fire contacts number n_{fc} (MAA that were attacked) by the time t of the enemy MAA strikes are repelled will be determined taking into account the time of SAMS is in the S_{10} occupied state and its "productivity" μ:

$$n_{fc}(t) = \mu \int_0^t P_{01}(\tau)\, d\tau =$$

$$= \frac{1}{P^*}\left(1 - \frac{1}{\alpha}\left[\lambda_1 e^{\lambda_2 \cdot \mu t} - \lambda_2 e^{\lambda_1 \cdot \mu t} \right] \right). \qquad (21)$$

To verify the simplest combat model (19), (20), (21), we will use equality (21) and find the maximum possible value of fire contacts mathematical expectation number for the entire time until the moment of SAMS is defeated:

$$n_{\text{fc}\infty} = \lim_{t\to\infty} n_{\text{fc}}(t) = \frac{1}{P^*}\left(1 - \frac{1}{\alpha}\cdot 0\right) = \frac{1}{P^*}. \qquad (22)$$

Note that the result (22) coincided with the previously obtained value (5) for this variable for one SAMS.

Next, we find the relative value $n_{\text{fc}}(t)$ of fire contacts mathematical expectation number for any moment in time t of the battle. To do this, we divide the left side of equation (21) into the left side of equation (22) and the right side of equation (21) into the right side of equation (22), we obtain:

$$\frac{n_{fc}(t)}{n_{fc\infty}}=\frac{1}{P^*}\left(1-\frac{1}{\alpha}\left[\lambda_1 e^{\lambda_2\cdot\mu t}-\lambda_2 e^{\lambda_1\cdot\mu t}\right]\right)\cdot\left(\frac{1}{P^*}\right)^{-1}; \quad (23)$$

$$n^*_{fc}(t)=1-\frac{1}{\alpha}\left[\lambda_1 e^{\lambda_2\cdot\mu t}-\lambda_2 e^{\lambda_1\cdot\mu t}\right]. \quad (24)$$

Let's find the relative value of SAMS defeated mathematical expectation number $n^*(t)$ at any point in time t during the battle:

$$n^*(t)=\frac{n_{d.S}(t)}{n_0}=\frac{n_{d.S}(t)}{1}=n_{d.S}(t)=$$
$$=0\cdot[P_{00}(t)+P_{01}(t)]+1\cdot P_{10}(t)=P_{10}(t). \quad (25)$$

Therefore, the relative value of the mathematical expectation of SAMS destroyed number at any point in time is equal to the probability of the state P_{10}:

$$n^*(t)=P_{10}(t)=1-\frac{1}{\alpha}\left[\lambda_1\cdot e^{\lambda_2\cdot\mu t}-\lambda_2\cdot e^{\lambda_1\cdot\mu t}\right]. \quad (26)$$

Comparing equations (24) and (26), we are convinced of their identity:

$$n^*(t)=n^*_{fc}(t). \quad (27)$$

Using equations (2), (6) and (8), it can be found that the expression for the relative value of enemy MAAs shot down mathematical expectation number $N^*_{sda}(t)$ differs from the expression $n^*_{fc}(t)$ for the probability of destroying a MAA during fire contact and at the same time coincides with equation (21):

$$N^*_{sda}(t)=\frac{N_{sda}(t)}{N_{sda\infty}}=\frac{P_{sda}\cdot n_{fc}(t)}{P_{sda}\cdot n_{fc\infty}}=\frac{n_{fc}(t)}{n_{fc\infty}}=n^*_{fc}(t). \quad (28)$$

Based on equations (22)–(28), it can be stated that in the description of the simplest anti-aircraft combat model (17)–(21) there is equality of the relative values of the mathematical expectations of SAMS fire contacts number and losses of the sides:

$$n^*(t)=N^*_{sda}(t)=n^*_{fc}(t). \quad (29)$$

Therefore, the developed simplest combat model of a single SAMS turns out to be adequate for real combat with the accuracy of the accepted hypotheses 1 and 2 regarding the most essential properties of anti-aircraft battle.

To roughly estimate the expected results of an anti-aircraft battle of an AAM grouping consisting of n_0 single-channel SAMS on an air target and repelling an enemy strike of MAA intensity I per minute, the grouping battle model can be replaced by a set of single SAMS battle models (17)–(21), each of which repels MAA strikes of intensity I_1:

$$I_1=\frac{I}{n_0}. \quad (30)$$

In this case, the total result can be defined as the sum of the single SAMS battles results. For the conditions of the considered example of an anti-aircraft battle of an AAM grouping (Figure 1, Table 1), the application of equality (26) makes it possible to find estimates of the battle results (Table 1, items 36–39), which differ from the real results within 0.6 %.

This degree of calculated comparison and real results allows us to consider equality (26) as a variant of the internal law's air combat analytical description – the equality of the mathematical expectations relative values of losses for the parties at any moment in time of the battle:

$$w(t)\approx n^*(t)=P_{10}(t)=1-\frac{1}{\alpha}\left[\lambda_1\cdot e^{\lambda_2\cdot\mu t}-\lambda_2\cdot e^{\lambda_1\cdot\mu t}\right] \quad (31)$$

where the elements of equality (31) are described in equality (20).

Conclusions

The use of modeling technology [6] elements made it possible to identify the most significant properties and find a description (31) of the internal law (12) of the anti-aircraft missile forces grouping real battle during the performance of AD tasks. At the same time, the forecast error using the specified law can be less than 0.6 % (Table 1, item 40), which allows this law to be applied for practical decision-making.

In addition, the use of technology [6] made it possible to select a mathematical apparatus statistically adequate to the combat processes, with the help of which it became possible to build a model of anti-aircraft combat of a single SAMS division and its application to predict the results of anti-aircraft combat of the AAM grouping.

The developed model describes the dynamics of the battle and has a clear analytical description of the found internal law of anti-aircraft combat, which makes the statement about its adequacy to real combat with the accuracy of the accepted hypotheses 1 and 2 regarding its most essential properties justified.

Therefore, there is reason to believe that the research objective has been achieved.

The direction of further research is seen as the construction of models that reflect the features indicated for the example of a real anti-aircraft battle of an AAM grouping, namely the appearance of MAA groups with an unknown (random) composition in the enemy air strike, as well as the location of SAMS on the terrain, which leads to the effect of incomplete enemy MAA availability for shelling by free SAMS groups due to the limited size of SAMS fire zones. In addition, a relevant direction is the development of a model that allows taking into account possible mutual assistance between SAMS divisions of the AAM grouping when firing at enemy MAA in their common anti-aircraft missile fire zone.

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МОДЕЛЮВАННЯ І ПРОГНОЗ РЕЗУЛЬТАТІВ ПРОТИПОВІТРЯНОГО БОЮ УГРУПОВАННЯ ЗЕНІТНИХ РАКЕТНИХ ВІЙСЬК

У ході військових дій захист важливих об’єктів держави та угруповань військ від ударів засобів повітряного противника покладається на зенітні ракетні війська (ЗРВ) у складі зенітних ракетних бригад, полків та підрозділів, які розгортаються у бойові порядки у позиційних районах і формують угруповання ЗРВ. Таке угруповання виконує своє бойове завдання під час протиповітряного бою як сукупності послідовних і одночасних протиповітряних боїв (вогневих контактів) зенітних ракетних *дивізіонів із засобами повітряного нападу (ЗПН) противника. Склад учасників, кількість та параметри вогневих контактів у просторі й часі бою, а також можливі втрати сторін заздалегідь не відомі і є випадковими величинами.*

Однак під час побудови бойового порядку угруповання ЗРВ завжди виникає практичне завдання прогнозу таких результатів і параметрів протиповітряного бою, як втрати ЗПН противника та зенітних ракетних комплексів угруповання, кількості вогневих контактів, достатності запасу зенітних керованих ракет та складу зенітних ракетних дивізіонів для відбиття першого і наступних ударів ЗПН противника.

Найнадійнішим є прогнозування, яке ґрунтується на внутрішніх законах досліджуваного процесу, що зумовило необхідність пошуку внутрішніх властивостей і закономірностей принципово неповторного у просторі, часі та у складі учасників протиповітряного бою угруповання ЗРВ.

Для такого пошуку було використано авторський спеціальний метод, а також відомий приклад реального бою. Метод дав змогу виявити основні властивості та внутрішній закон бою; вибрати статистично обґрунтований математичний апарат моделювання у класі марковських процесів із безперервним часом та дискретними станами; розробити необхідну модель з аналітичним описом внутрішнього закону бою; перевірити її адекватність на прикладі реального протиповітряного бою; оцінити точність прогнозування з використанням розробленої моделі.

Ключові слова: *протиповітряна оборона, угруповання зенітних ракетних військ, марковські процеси, моделювання.*

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